

Semester 3 Mathematics Minor (Vector Space)

Question Bank

Module 1: Vector spaces and subspaces

1. Define Vector Space and state its properties.
2. Check whether the following is vector space or not?
 - (a) Consider $S = \{(x, y, 2) / x, y \in R\}$ over R . [No]
 - (b) Consider $S = \{(x, x^2) / x \in R\}$ over R . [No]
 - (c) Consider $S = \left\{ \begin{bmatrix} a & b \\ c & a+2 \end{bmatrix} \right\}$ s.t. $a, b, c \in R$ $M_{2 \times 2}(R)$. [No]
3. Consider $W = \{(x, y, z) \text{ s.t. } z = x + y \text{ where } x, y, z \in R\}$ a subset of R^3 . Show that W is a subspace of R^3 .
4. Prove the following: The Intersection of two subspaces of a Vector Space V is a subspace of V .
5. Check whether the Union of two subspaces of a Vector Space V is a subspace of V or not. [No]

Module 2: System of linear equations, Linear combination, Basis of vector spaces

6. Express the vector $(4, 2)$ as a linear combination of vectors $(1, 1)$ and $(2, -3)$. [16/5, 2/5]
7. Express the polynomial $1 + 2x + x^2$ in $P_2[X]$ as a linear combination of $1 + x, x + x^2, x^2$ in $P_2[X]$ [1,1,0]
8. Express the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ in $M_{2 \times 2}(R)$ as linear combination of following matrices

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}.$$
 [0, 1, 0, 1]
9. Check whether the following is Linearly Dependent or Linearly Independent Set.
 - (a) $\{(1, 0, 0), (1, 1, 1), (1, 2, 0)\}$ in R^3 [L.I.]
 - (b) $\{(1, 0, 0), (1, 1, 1), (0, 1, 0), (0, 0, 1)\}$ in R^3 [L.D.]
10. Define Basis of Vector space and show the following:
 - (a) Set $\{(1, 0), (0, 1)\}$ is Basis of R^2
 - (b) Set $\{(1, 0, 0), (1, 1, 0), (1, 1, 2)\}$ forms Basis of R^3
 - (c) Set of Polynomials $\{1, 2 + x, 3x^2\}$ is basis of $P_2[x]$

11. $A = \begin{bmatrix} x & x & 2 \\ 2 & x & x \\ x & 2 & x \end{bmatrix}$ is a matrix over R . Find all possible values of x for which

(a) Rank $A = 1$

(b) Rank $A = 2$

[(a) 2, (b) -1]

12. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & x & 1 \\ -1 & -1 & x \end{bmatrix}$, show that the rank of A is always 3 for any value of $x \in R$.

13. Find Rank A by Row Reduction Method.

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 8 \\ 2 & 4 & 7 & 6 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$, (c) $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, (d) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{bmatrix}$, (e) $\begin{bmatrix} -4 & -5 & 6 \\ -1 & 2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$

[(a) 3, (b) 4, (c) 3, (d) 3, (e) 2]